Unquenched quark model for baryons: magnetic moments, spins and orbital angular momenta

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We present an unquenched quark model for baryons in which the effects of the quark-antiquark pairs ($u\bar{u}$, $d\bar{d}$ and $s\bar{s}$) are taken into account in an explicit form via a microscopic, QCD-inspired, quark-antiquark creation mechanism. In the present approach, the contribution of the quark-antiquark pairs can be studied for any inital baryon and for any flavor of the $q\bar{q}$ pairs. It is shown that, while the inclusion of the $q\bar{q}$ pairs does not affect the baryon magnetic moments, it leads to a sizeable contribution of the orbital angular momentum to the spin of the proton and the Λ hyperon.

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I. INTRODUCTION

One of the main goals of hadronic physics is to understand the structure of the nucleon and its excited states in terms of effective degrees of freedom and, at a more fundamental level, the emergence of these effective degrees of freedom from QCD, the underlying theory of quarks and gluons [1]. Despite the progress made in lattice calculations, it remains a daunting problem to solve the QCD equations in the non-perturbative region. Therefore, one has developed effective models of hadrons, such as bag models, chiral quark models, soliton models [2], instanton liquid model [3] and the constituent quark model. Each of these approaches is constructed in order to mimic some selected properties of the strong interaction, but obviously none of them is QCD.

An important class is provided by constituent quark models (CQM) which are based on constituent (effective) quark degrees of freedoms. There exists a large variety of CQMs, among others the Isgur-Karl model [4], the Capstick-Isgur model [5], the collective model [6], the hypercentral model [7], the chiral boson-exchange model [8] and the Bonn instanton model [9]. While these models display important and peculiar differences, they share the main features: the effective degrees of freedom of three constituent quarks (qqq configurations), the $SU(6)$ spin-flavor symmetry and a long-range confining potential. Each of these models reproduce the mass spectrum of baryon resonances reasonably well, but at the same time, they show very similar deviations for other observables, such as photocouplings, helicity amplitudes and strong decays. As an example, we mention helicity amplitudes (or transition form factors) which typically show deviations from CQM calculations at low values of Q^2 (see Fig. 1 for the $D_{13}(1520)$ resonance). The problem of

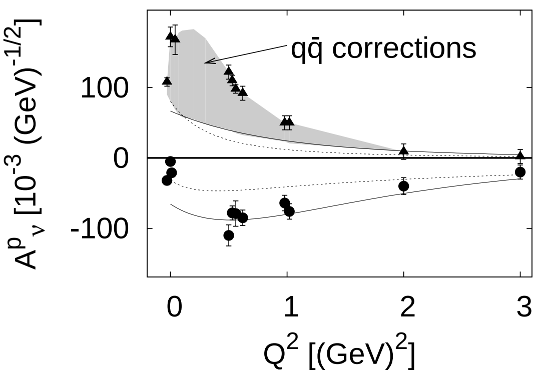

Figure 1: The helicity amplitudes as a function of Q^2 for the $D_{13}(1520)$ resonance. Experimental data [10] are compared with theoretical predictions from the collective $U(7)$ model [6] (dotted line) and the hypercentral model [7] (solid line). The dashed line corresponds to a fit to the experimental data.

missing strength at low Q^2 in constituent quark model calculations indicates that some fundamental mechanism is lacking in the dynamical description of hadronic structure. This mechanism can be identified with the production of quark-antiquark pairs [11, 12]. Low values of Q^2 correspond to a distance scale at which there is a higher probability of string breaking and thus of quark-antiquark pair production.

Additional evidence for higher Fock components in the baryon wave function ($qqq - q\bar{q}$ configurations) comes from CQM studies of the strong decays of baryon resonances, that are on average underpredicted by CQMs [6, 13]. More direct indications for the importance of quark-antiquark components in the proton come from measurements of the $\bar{d}/\bar{u}$ asymmetry in the nucleon sea [14, 15] and parity-violating electron scattering experiments which report a nonvanishing strange quark contribution, albeit (very) small, to the charge and magnetiza-

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tion distributions [16, 17].

The role of higher Fock components in baryon wave functions has been studied by many authors in the context of meson cloud models, such as the cloudy bag model, meson convolution models and chiral models [14, 18]. In these models, the flavor asymmetry of the proton can be understood in terms of couplings to the pion cloud. There have also been several attempts to study the importance of higher Fock components in the context of the constituent quark model. In this respect we mention the work by Riska and coworkers who introduce a small number of selected higher Fock components which are then fitted to reproduce the experimental data [19]. However, these studies lack an explicit model or mechanism for the mixing between the valence and sea quarks. The Rome group studied the pion and nucleon electromagnetic form factors in a Bethe-Salpeter approach, mainly thanks to the dressing of photon vertex by means of a vector-meson dominance parametrization [20]. Koniuk and Guiasu used a convolution model with CQM wave functions and an elementary emission model for the coupling to the pion cloud to calculate the magnetic moments and the helicity amplitudes from the nucleon to the Δ resonance [21]. It was found that the nucleon magnetic moments were unchanged after renomalization of the parameters, but that the missing strength in the helicity amplitudes of the Δ could not be explained with pions only.

The impact of $q\bar{q}$ pairs in hadron spectroscopy was originally studied by Törnqvist and Zenczykowski in a quark model extended by the $^3P_0$ model [22]. Even though their model only includes a sum over ground state baryons and ground state mesons, the basic idea of the importance to carry out a sum over a complete set of intermediate states was proposed in there. Subsequently, the effects of hadron loops in mesons was studied by Geiger and Isgur in a flux-tube breaking model in which the $q\bar{q}$ pairs are created in the $^3P_0$ state with the quantum numbers of the vacuum [23, 24, 25]. In this approach, the quark potential model arises from an adiabatic approximation to the gluonic degrees of freedom embodied in the flux-tube [26]. It was shown that cancellations between apparently uncorrelated sets of intermediate states occur in such a way that the modification in the linear potential can be reabsorbed, after renormalization, in the new strength of the linear potential [24]. In addition, the quark-antiquark pairs do not destroy the good CQM results for the mesons [24] and preserve the OZI hierarchy [25] provided that the sum be carried out over a large tower of intermediate states. A first application of this procedure to baryons was presented in [27] in which the importance of $s\bar{s}$ loops in the proton were studied by taking into account the contribution of the six diagrams of Fig. 2 in combination with harmonic oscillator wave functions for the baryons and mesons and a $^3P_0$ pair creation mechanism. This approach has the advantage that the effects of quark-antiquark pairs are introduced explicitly via a QCD-inspired pair-creation mechanism, which opens the possibility to study the importance of $q\bar{q}$ pairs in baryons and mesons in a systematic and unified way.

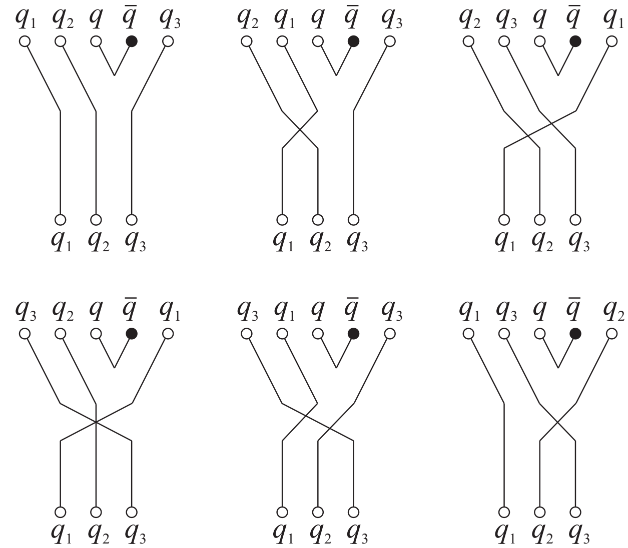

Figure 2: Quark line diagrams for $A \rightarrow BC$ with $q\bar{q} = s\bar{s}$ and $q_1q_2q_3 = uud$.

The aim of this article is to present an unquenched quark model, valid for any initial baryon (or baryon resonance), any flavor of the quark-antiquark pair (not only $s\bar{s}$, but also $u\bar{u}$ and $d\bar{d}$ loops) and any CQM. In order to test the consistency of the formalism we first calculate the baryon magnetic moments which constitute one of the early successes of the CQM. Finally, we study an application of the unquenched quark model to the spin of the proton and the Λ hyperon, and calculate in explicit form the contributions of the valence and sea quark spins and the orbital angular momentum. Preliminary results of this work were presented in various conference proceedings [28, 29, 30].

II. UNQUENCHED QUARK MODEL

In this section, we present a procedure for unquenching the quark model in which the effects of quark-antiquark pairs are introduced explicitly into the CQM via a QCD-inspired $^3P_0$ pair-creation mechanism. The present approach is motivated by the work of Isgur and coworkers on the flux-tube breaking model in which they showed that the CQM emerges as the adiabatic limit of the flux-tube model to which the effects of $q\bar{q}$ pair creation are added as a perturbation [27]. Our approach is based on a CQM to which the quark-antiquark pairs with vacuum quantum numbers are added as a perturbation. The pair-creation mechanism is inserted at the quark level and the one-loop diagrams are calculated by summing over all possible intermediate states.

Under these assumptions, the baryon wave function

consists of a zeroth order three-quark configuration plus a sum over all possible higher Fock components due to the creation of $^3P_0$ quark-antiquark pairs. To leading order in pair creation, the baryon wave function can be written as

$$\mid \psi_A \rangle \;=\; \mathcal{N} \left[\mid A \rangle + \sum_{BClJ} \int d\vec{k} \; \mid BC\vec{k}\, lJ \rangle \right. \left. \times \frac{\langle BC\vec{k}\, lJ \mid T^{\dagger} \mid A \rangle}{M_A - E_B - E_C} \right] \;, \quad (1)$$

where $T^{\dagger}$ is the $^3P_0$ quark-antiquark pair creation operator [31], A is the baryon, B and C represent the intermediate baryon and meson, and M_A, E_B and E_C are their respective energies, $\vec{k}$ and l the relative radial momentum and orbital angular momentum of B and C, and J is the total angular momentum $\vec{J} = \vec{J}_B + \vec{J}_C + \vec{l}$.

The $^3P_0$ quark-antiquark pair-creation operator, $T^{\dagger}$, can be written as [31]

$$T^{\dagger} \;=\; -3\,\gamma_0 \int d\vec{p}_4 \, d\vec{p}_5 \, \delta(\vec{p}_4 + \vec{p}_5) \, C_{45} \, F_{45} \, e^{-r_q^2 (\vec{p}_4 - \vec{p}_5)^2/6} \left[\chi_{45} \, \times \, \mathcal{Y}_1(\vec{p}_4 - \vec{p}_5) \right]_0^{(0)} \, b_4^{\dagger}(\vec{p}_4) \, d_5^{\dagger}(\vec{p}_5) \;. \quad (2)$$

Here, $b_4^{\dagger}(\vec{p}_4)$ and $d_5^{\dagger}(\vec{p}_5)$ are the creation operators for a quark and an antiquark with momenta $\vec{p}_4$ and $\vec{p}_5$, respectively. The quark and antiquark pair is characterized by a color singlet wave function C_{45}, a flavor singlet wave function F_{45}, a spin triplet wave function χ_{45} with spin $S = 1$ and a solid spherical harmonic $\mathcal{Y}_1(\vec{p}_4 - \vec{p}_5)$ that indicates that the quark and antiquark are in a relative P wave. The operator $T^{\dagger}$ creates a pair of constituent quarks with an effective size, thus the pair creation point is smeared out by a gaussian factor whose width r_q was determined from meson decays to be approximately $0.25 - 0.35$ fm [25, 27, 32]. In our calculations, we take an average value, $r_q = 0.30$ fm. The dimensionless constant γ_0 is the intrinsic pair-creation strength which was determined from strong decays of baryons as $\gamma_0 = 2.60$ [13]. The matrix elements of the pair-creation operator $T^{\dagger}$ were derived in explicit form in the harmonic oscillator basis [31].

In this paper, we use the harmonic oscillator limit of algebraic models of hadron structure [6, 33] to calculate the baryon and meson energies appearing in the denominator of Eq. (1). In these algebraic models, the mass operators for baryons and mesons consist of a harmonic oscillator term and a Gürsey-Radicati term which reproduces the splitting of the $SU(6)$ multiplets without mixing the harmonic oscillator wave functions. As a consequence, the baryon and meson wave functions have good flavor symmetry and depend on a single oscillator parameter which, following [27], is taken to be $\hbar\omega_{\rm baryon} = 0.32$ GeV for the baryons and $\hbar\omega_{\rm meson} = 0.40$ GeV for the mesons.

The matrix elements of an observable $\hat{\mathcal{O}}$ can be calculated as

$$\mathcal{O} = \langle \psi_A \mid \hat{\mathcal{O}} \mid \psi_A \rangle = \mathcal{O}_{\rm valence} + \mathcal{O}_{\rm sea} \;, \quad (3)$$

where the first term corresponds to the contribution of the three valence quarks and the second to the higher Fock components, *i.e.* the presence of the quark-antiquark pairs.

In order to calculate the effects of quark-antiquark pairs on an observable, one has to evaluate the sum over all possible intermediate states in Eq. (1). The sum over intermediate meson-baryon states includes for baryons all radial and orbital exications up to a given oscillator shell combined with all possible $SU(6)$ spin-flavor multiplets, and for mesons all radial and orbital excitations up to given oscillator shell and all possible nonets. This problem was solved by means of group theoretical techniques to construct an algorithm to generate a complete set of intermediate meson-baryon states in spin-flavor space for an arbitrary oscillator shell. This property makes it possible to perform the sum over intermediate states up to saturation and not only for the first few shells as in [27]. In addition, it allows the evaluation of the contribution of quark-antiquark pairs for any initial baryon $q_1q_2q_3$ (ground state or resonance) and for any flavor of the $q\bar{q}$ pair (not only $s\bar{s}$, but also $u\bar{u}$ and $d\bar{d}$), and for any model of baryons and mesons, as long as their wave functions are expressed in the basis of harmonic oscillator wave functions.

III. CLOSURE LIMIT

Before discussing an application of the unquenched model to baryon magnetic moments and spins, we study the so-called closure limit in which the intermediate states appearing in Eq. (1) are degenerate in energy and hence the energy denominator becomes a constant independent of the quantum numbers of the intermediate states. In the closure limit, the evaluation of the contribution of the quark-antiquark pairs (or the higher Fock components) simplifies considerably, since the sum over intermediate states can be solved by closure and the contribution of the quark-antiquark pairs to the matrix element reduces to

$$\mathcal{O}_{\rm sea} \;\propto\; \langle A \mid T \, \hat{\mathcal{O}} \, T^{\dagger} \mid A \rangle \;. \quad (4)$$

Since the $^3P_0$ pair-creation operator of Eq. (2) is a flavor singlet and the energy denominator in Eq. (1) is reduced to a constant in the closure limit, the higher Fock component of the baryon wave function has the same flavor symmetry as the valence quark configuration $\mid A \rangle$. Moreover, if the pair-creation operator does not couple to the motion of the valence quarks, the valence quarks act as spectators. In this case, the contribution of the $q\bar{q}$ pairs simplifies further to the expectation value of $\hat{\mathcal{O}}$ between the $^3P_0$ pair states created by $T^{\dagger}$

$$\mathcal{O}_{\rm sea} \;\propto\; \langle 0 \mid T \, \hat{\mathcal{O}} \, T^{\dagger} \mid 0 \rangle \;, \quad (5)$$

the so-called closure-spectator limit [27] which is a special case of the closure limit.

Table I: Δu, Δd and Δs for ground state octet baryons in the closure limit in units of $(\Delta u)_p/4$.

qqq	$^2 8[56,0^+]$	Δu	Δd	Δs
uud	p	4	-1	0
udd	n	-1	4	0
uus	Σ^+	4	0	-1
uds	Σ^0	2	2	-1
	Λ	0	0	3
dds	Σ^-	0	4	-1
uss	Ξ^0	-1	0	4
dss	Ξ^-	0	-1	4

As an example, we discuss the contribution of the quark-antiquark pairs for the operator $2[s_z(q)+s_z(\bar{q})]$ in the closure limit

$$\Delta q = 2\,\langle s_z(q)+s_z(\bar{q})\rangle \ . \tag{6}$$

Δq is the nonrelativistic limit of the axial charges and denotes the fraction of the baryon's spin carried by quarks and antiquarks with flavor $q = u,\ d,\ s$. In Table I we present the results for the ground state octet baryons with $^2 8[56,0^+]_{1/2}$. Since the valence-quark configuration of the proton and the neutron does not contain strange quarks, the valence quarks act as spectators in the calculation of Δs. Therefore, the contribution of Δs to the spin of the nucleon is given by the closure-spectator limit which vanishes due to the symmetry properties of the operator Δs and the $^3P_0$ wave function. The same holds for the contribution of $d\bar{d}$ pairs to the Σ^+ and Ξ^0 hyperons, and that of $u\bar{u}$ pairs to the Σ^- and Ξ^- hyperons. The vanishing contributions of Δu and Δd to the spin of the Λ hyperon are a consequence of the Λ wave function in which the up and down quarks are coupled to isospin and spin zero. Similarly, the vanishing contributions of Δq to the spin of the ground state decuplet baryons with $^4 10[56,0^+]_{3/2}$ in Table II can be understood in the closure-spectator limit.

In addition, since in the closure limit the baryon wave function has the same flavor symmetry as the valence quark configuration, it can be shown that the flavor dependence of the contribution of the quark-antiquark pairs to the spin of the ground state baryons in Tables I and II is the same as that of the valence quarks

$$\Delta u_{\rm sea} : \Delta d_{\rm sea} : \Delta s_{\rm sea} = \Delta u_{\rm val} : \Delta d_{\rm val} : \Delta s_{\rm val} \ . \tag{7}$$

The results for octet and decuplet ground state baryons are related by

$$(\Delta u + \Delta d + \Delta s)_{\rm dec} = 3\,(\Delta u + \Delta d + \Delta s)_{\rm oct} \ . \tag{8}$$

The same relation holds for the orbital angular momentum

$$(\Delta L)_{\rm dec} = 3\,(\Delta L)_{\rm oct} \ , \tag{9}$$

Table II: As Table I, but for ground state decuplet baryons.

qqq	$^4 10[56,0^+]$	Δu	Δd	Δs
uuu	Δ^{++}	9	0	0
uud	Δ^+	6	3	0
udd	Δ^0	3	6	0
ddd	Δ^-	0	9	0
uus	$\Sigma^{*\,+}$	6	0	3
uds	$\Sigma^{*\,0}$	3	3	3
dds	$\Sigma^{*\,-}$	0	6	3
uss	$\Xi^{*\,0}$	3	0	6
dss	$\Xi^{*\,-}$	0	3	6
sss	Ω^-	0	0	9

with

$$\Delta L = \sum_q \Delta L(q) = \sum_q \langle l_z(q) + l_z(\bar{q})\rangle \ . \tag{10}$$

Note that, even if the valence quark configuration $[56,0^+]$ does not carry orbital angular momentum, there is a nonzero contribution of the quark-antiquark pairs in the closure limit, albeit small (less than 1 %) in comparison with that of the quark spins. Obviously, the sum of the spin and orbital parts gives the total angular momentum of the baryon

$$J = \frac{1}{2}\Delta\Sigma + \Delta L \ , \tag{11}$$

with

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s \ . \tag{12}$$

At a qualitative level, the closure limit helps to explain the phenomenological success of the CQM because the $SU(3)$ flavor symmetry of the baryon wave function is preserved. As an example, the strange content of the proton vanishes in the closure-spectator limit due to many cancelling contributions in the sum over intermediate states in Eq. (1). Away from the closure limit, the strangeness content of the proton is expected to be small, in agreement with the experimental data from parity-violating electron scattering (for some recent data see [16, 17]). Even though in this case the cancellations are no longer exact, many intermediate states contribute with opposite signs, and the net result is nonzero, but small. This means that even if the flavor symmetry of the CQM is broken by the higher Fock components, the net results are still to a large extent determined by the flavor symmetry of the valence quark configuration. Similar arguments were applied to the preservation of the OZI hierarchy in the context of the flux-tube breaking model [25]. Therefore, the closure limit not only provides simple expressions for the relative flavor content of physical observables, but also gives further insight into the origin

of cancellations between the contributions from different intermediate states.

In addition, the results in closure limit in Tables I and II impose very stringent conditions on the numerical calculations, since each entry involves the sum over all possible intermediate states. Therefore, the closure limit provides a highly nontrivial test of the computer codes which involves both the spin-flavor sector, the permutation symmetry, the construction of a complete set of intermediate states in spin-flavor space for each radial excitation and the implementation of the sum over all of these states.

In this section, we discussed some qualitative properties of the unquenched quark model in the closure limit. In the following sections, we study the effects of quark-antiquark pairs on the magnetic moments and the spin of octet baryons in the general case, *i.e.* beyond the closure limit.

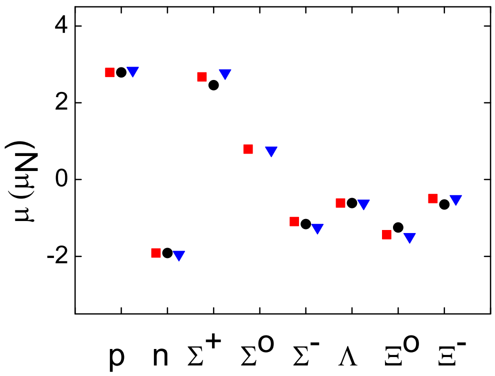

Figure 3: (color online) Magnetic moments of octet baryons: experimental values from PDG [34] (circles), CQM (squares) and unquenched quark model (triangles).

IV. MAGNETIC MOMENTS

The unquenching of the quark model has to be carried out in such a way as to preserve the phenomenological successes of the constituent quark model. It is well known that the CQM gives a good description of the baryon magnetic moments, even in its simplest form in which the baryons are treated in terms of three constituent quarks in a relative S-wave. The quark magnetic moments are determined by fitting the magnetic moments of the proton, neutron and Λ hyperon to give $\mu_u = 1.852$, $\mu_d = -0.972$ and $\mu_s = -0.613\ \mu_N$ [34].

In the unquenched CQM the baryon magnetic moments also receive contributions from the quark spins of the pairs and the orbital motion of the quarks

$$\vec{\mu} = \sum_q \mu_q \left[2\vec{s}(q) + \vec{l}(q) - 2\vec{s}(\bar{q}) - \vec{l}(\bar{q}) \right] \ , \qquad (13)$$

where $\mu_q = e_q\hbar/2m_qc$ is the quark magnetic moment. In Fig. 3 we show a comparison between the experimental values of the magnetic moments of the octet baryons (circles) and the theoretical values obtained in the CQM (squares) and in the unquenched quark model (triangles). The results for the unquenched quark model were obtained in a calculation involving a sum over intermediate states up to five oscillator shells for both baryons and mesons. We note, that the results for the magnetic moments, after renormalization, are almost independent on the number of shells included in the sum over intermediate states. The values of the magnetic moments in the unquenched quark model are very similar to those in the CQM. The largest difference is observed for the charged Σ hyperons, but the relation between the magnetic moments of Σ hyperons [35], $\mu(\Sigma^0) = [\mu(\Sigma^+) + \mu(\Sigma^-)]/2$, is preserved in the unquenched calculation due to isospin symmetry.

The inclusion of the $q\bar{q}$ pairs leads to slightly different values of the quark magnetic moments, $\mu_u = 2.066$, $\mu_d = -1.110$ and $\mu_s = -0.633\ \mu_N$ as for the CQM. This is related to the well-known phenomenon, that a calculation carried out in a truncated basis leads to effective parameters in order to reproduce the results obtained in a more extended basis. The results in the unquenched quark model are practically identical, after renormalization, to the ones in the CQM, which shows that the addition of the quark-antiquark pairs preserves the good CQM results for the baryon magnetic moments. A similar feature was found in the context of the flux-tube breaking model for mesons in which it was shown that the inclusion of quark-antiquark pairs preserved the linear behavior of the confining potential as well as the OZI hierarchy [25]. The change in the linear potential caused by the bubbling of the pairs in the string could be absorbed into a renormalized strength of the linear potential.

The results for the magnetic moments can be understood qualitatively in the closure limit in which the relative contribution of the quark spins from the quark-antiquark pairs is the same as that from the valence quarks. Moreover, since in the closure limit the contribution of the orbital angular momentum is small in comparison to that of the quark spins, the results for the baryon magnetic moments are almost indistinguishable from those of the CQM. Away from the closure limit, even though the relations between the different contributions no longer hold exactly, they are still valid approximately. In addition, there is now a contribution from the orbital part (at the level of ~ 5 %) which is mainly due to the baryon-pion channel.

In summary, the inclusion of the effects of quark-antiquark pairs preserves, after renormalization, the good results of the CQM for the magnetic moments of the octet baryons.

Table III: Contribution of Δu, Δd , Δs, $\Delta\Sigma = \Delta u + \Delta d + \Delta s$ and ΔL to the proton spin in the unquenched quark model (UCQM).

				UCQM		
p	CQM	EJS	DIS	val	sea	total
Δu	4/3	0.928	0.842	0.504	0.594	1.098
Δd	–1/3	–0.342	–0.427	–0.126	–0.291	–0.417
Δs	0	0.000	–0.085	0.000	–0.005	–0.005
$\Delta\Sigma$	1	0.586	0.330	0.378	0.298	0.676
$2\Delta L$	0	0.414		0.000	0.324	0.324
$2J$	1	1.000		0.378	0.622	1.000

V. SPINS AND ORBITAL ANGULAR MOMENTA

In this section, we discuss an application of the unquenched quark model to the spin content of the proton and the Λ hyperon. Ever since the European Muon Collaboration at CERN showed that *the total quark spin constitutes a rather small fraction of the spin of the nucleon* [36], there has been an enormous interest in the spin structure of the proton [37, 38, 39]. The original EMC result suggested that the contribution of the quark spins was close to zero, $\Delta\Sigma = 0.120 \pm 0.094 \pm 0.138$ [36]. Thanks to a new generation of experiments and an increase in experimental accuracy, the fraction of the proton spin carried by the quarks and antiquarks is now known to be about one third. The most recent values were obtained by the HERMES and COMPASS collaborations, $\Delta\Sigma = 0.330 \pm 0.011 \pm 0.025 \pm 0.028$ at $Q^2 = 5$ GeV^2 [40] and $0.33 \pm 0.03 \pm 0.05$ at $Q^2 = 3$ GeV^2 [41], respectively. The EMC results led to the idea that the proton might contain a substantial amount of polarized glue which could contribute to reducing the contribution of the quark spins through the $U(1)$ axial anomaly [42]. Therefore, much of the early theoretical work was in the direction of understanding the role of polarized gluons and the axial anomaly to resolve the puzzle of the proton spin [37, 42, 43]. However, there is increasing evidence from recent experiments, that at low values of Q^2 the gluon contribution is rather small (either positive or negative) and compatible with zero [44, 45], which rules out the possibility that most of the missing spin be carried by the gluon. At the same time, this indicates that the missing spin of the proton has to be attributed to others mechanisms [38, 39], in particular to the orbital angular momentum of the quarks and antiquarks [29, 46, 47, 48].

A. Proton spin

The formalism developed in Section II makes it possible to study the effect of quark-antiquark pairs on the fraction of the proton spin carried by the quark (antiquark) spins and orbital angular momentum by means of an explicit calculation in an unquenched quark model. Just as in other effective models [38, 49, 50] the unquenched quark model does not include gluonic effects associated with the axial anomaly, and therefore the contribution from the gluons is missing from the outset. The total spin of the proton can be written as

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta L = \frac{1}{2}\left(\Delta u + \Delta d + \Delta s\right) + \Delta L \, . \quad (14)$$

The axial charges,

$$\Delta q = \langle p \uparrow | \bar{q}\gamma_z\gamma_5 q | p \uparrow \rangle \ , \quad (15)$$

denote the fraction of the proton's spin carried by the light quarks and antiquarks with flavor $q = u,\, d,\, s$. In the nonrelativistic limit, they are given by the matrix elements

$$\Delta q = 2 \left\langle p \uparrow | s_z(q) + s_z(\bar{q}) | p \uparrow \right\rangle . \quad (16)$$

The last term in Eq. (14) represents the contribution from orbital angular momentum

$$\Delta L = \sum_q \Delta L(q) = \sum_q \left\langle p \uparrow | l_z(q) + l_z(\bar{q}) | p \uparrow \right\rangle \ . \quad (17)$$

In the present unquenched quark model, the $SU(3)$ flavor symmetry is satisfied by the valence quark configuration, but broken by the quark-antiquark pairs. In the unquenched calculation we use harmonic oscillator wave functions up to five oscillator shells for both the intermediate baryons and mesons. As mentioned in Section II, all parameters were taken from the literature [13, 27]. No attempt was made to optimize their values in order to improve the agreement with experimental data.

Table III shows that the inclusion of the quark-antiquark pairs has a dramatic effect on the spin content of the proton. Whereas in the CQM the proton spin is carried entirely by the (valence) quarks, it is shown in Table III that in the unquenched calculation 67.6 % is carried by the quark and antiquark spins and the remaining 32.4 % by orbital angular momentum. The orbital angular momentum due to the relative motion of the baryon with respect to the meson accounts for 31.7 % of the proton spin, whereas the orbitally excited baryons and mesons in the intermediate state only contribute 0.7 %. Finally we note, that the orbital angular momentum arises almost entirely from the relative motion of the nucleon and Δ resonance with respect to the π-meson in the intermediate states. In the closure limit, all mesons (including the pion) have the same mass and their contributions to the orbital angular momentum average out and reduce to less than 1 % of the proton spin.

On the contrary, the contribution of the quark and antiquark spins to the proton spin is dominated by the intermediate vector mesons. Since in the case of the quark spins the convergence of the sum over intermediate states is slow, we carried out the sum over five oscillator shells

for both the intermediate baryons and mesons. For each oscillator shell the sum is performed over a complete set of spin-flavor states. It is important to note that the contributions of the valence quark spins, the sea quark spins and the orbital angular momentum to the proton spin, 37.8 %, 29.8 % and 32.4 %, respectively, are comparable in size.

In the unquenched quark model there is a large contribution ($\sim$ 32 %) of orbital angular momentum to the proton spin, while for the proton magnetic moment it is relatively small ($\sim$ 5 %). This can be understood qualitatively from the difference in relative signs between the quark and antiquark contributions in Eqs. (13) for the magnetic moment and those in Eqs. (16,17) for the proton spin.

The present results for the singlet axial charge $a_0 = \Delta\Sigma$ are in qualitative agreement with the cloudy bag model and the Nambu-Jona-Lasinio model in which one finds $a_0 = 0.60$ [49] and 0.56 [50], respectively. The inclusion of kaon loops gives in both models a small value of the contribution of strange quarks, $\Delta s = -0.003$ and -0.006, respectively, in agreement with the unquenched calculations. Another effect of the quark-antiquark pairs is a reduction of the triplet and octet axial charges from their CQM values of 5/3 and 1 to $a_3 = \Delta u - \Delta d = 1.515$ and $a_8 = \Delta u + \Delta d - 2\Delta s = 0.681$, respectively. It is interesting to note that the ratio of these axial charges in the unquenched quark model is calculated to be $a_3/a_8 = 2.22$ which is very close to the value of 2.15 determined from hyperon semileptonic decays, but very different from the naive CQM value 5/3.

Experimentally, the contributions of the quark spins Δu, Δd and Δs to the spin of the proton are obtained by combining data from hyperon β decays and deep-inelastic lepton-nucleon scattering processes. First, the hyperon β decays $n \rightarrow p + e^- + \bar{\nu}_e$ and $\Sigma^- \rightarrow n + e^- + \bar{\nu}_e$ are used in combination with the assumption of $SU(3)$ flavor symmetry to determine the couplings $F = (a_3 + a_8)/4$ and $D = (3a_3 - a_8)/4$. Next, $\Delta\Sigma$ can be extracted from deep-inelastic electron-proton scattering experiments. As a result, Δq of the proton is given by

$$
\begin{aligned}
(\Delta u)_p &= \frac{1}{3}\left(\Delta\Sigma + 3F + D\right) \ , \\
(\Delta d)_p &= \frac{1}{3}\left(\Delta\Sigma - 2D\right) \ , \\
(\Delta s)_p &= \frac{1}{3}\left(\Delta\Sigma - 3F + D\right) \ . \qquad (18)
\end{aligned}
$$

The theoretical uncertainty in determining the values of F and D by assuming flavor symmetry were estimated to be of the order of 10-15 % [53, 57, 58, 59]. It is important to keep in mind that, even though the effect of flavor symmetry breaking on the hyperon decays may not be so large, for other quantities like $\Delta\Sigma$ and Δs it is much stronger [53, 57, 60]. The results of the HERMES analysis are presented in the column labeled DIS of Table III. These values were obtained by combining the couplings $F = 0.464$ and $D = 0.806$ as determined from hyperon β decays with $\Delta\Sigma = 0.330$ as extracted from the first moment of the spin structure function g_1^p [40]. For the purpose of reference, we also present the values for the naive quark model (CQM) which correspond to $F = 2/3$ and $D = \Delta\Sigma = 1$ and for the Ellis-Jaffe-Sehgal analysis (EJS), in which it is assumed that there are no polarized strange quarks in the proton [51, 52]. In the latter case, the spin content is calculated by using F and D from hyperon β decays and $\Delta\Sigma = 3F - D$. The remainder of the proton spin $1 - 3F + D$ is attributed to orbital angular momentum [46].

The importance of orbital angular momentum to the proton spin was discussed many years ago by Sehgal [46] in the context of the quark-parton model. Table III shows, that the results of the unquenched quark model are similar to those of the EJS analysis. More recently, Myhrer and Thomas emphasized the importance of spin and orbital angular momentum in the proton in the bag model [48] and discussed three effects that can convert quark spin into orbital angular momentum: the relativistic motion of the valence quarks, the one-gluon exchange corrections and the pion cloud of the nucleon. The contribution of the quark spins was estimated in a qualitative way to be in the range $0.35 < \Delta\Sigma < 0.40$.

Table IV: As Fig. III, but for the Λ hyperon.

				UCQM		
Λ	CQM	EJS	DIS	val	sea	total
Δu	0	–0.073	–0.159	0.000	–0.055	–0.055
Δd	0	–0.073	–0.159	0.000	–0.055	–0.055
Δs	1	0.733	0.647	0.422	0.539	0.961
$\Delta\Sigma$	1	0.586	0.330	0.422	0.429	0.851
$2\Delta L$	0	0.414		0.000	0.149	0.149
$2J$	1	1.000		0.422	0.578	1.000

B. Λ spin

The recent studies of the spin structure of the proton have raised a lot of questions about the importance of valence and sea quarks, gluons and orbital angular momentum. In this respect it is interesting to investigate the spin structure of other hadrons. The Λ hyperon is of special interest, since its polarization can be measured from the nonleptonic decay $\Lambda \rightarrow p\pi$ [52]. In addition, in the naive CQM its spin content resides entirely on the strange quark, $(\Delta u)_\Lambda = (\Delta d)_\Lambda = 0$ and $(\Delta s)_\Lambda = 1$, which makes it a clean example to study the spin structure of baryons. An investigation of the spin structure of the Λ hyperon is not only interesting in its own right, but also may shed light on the spin crisis of the proton.

Table IV shows that the unquenched quark model gives rise to a negatively polarized sea of up and down quarks. The contribution of quark spins for the Λ is found to be

larger than that for the proton, $(\Delta\Sigma)_\Lambda > (\Delta\Sigma)_p$.

It is interesting to compare the unquenched results with those of some previous analyses. In most other studies one had to make additional assumptions about the sea quarks in order to get an estimate of the spin content of the Λ hyperon in most. Under the assumption of $SU(3)$ flavor symmetry, the spin content of the octet baryons can be expressed in terms of that of the proton as [52, 68]

$$\begin{aligned}(\Delta u)_\Lambda = (\Delta d)_\Lambda &= \frac{1}{6}\left(\Delta u + 4\Delta d + \Delta s\right)_p \\ &= \frac{1}{3}\left(\Delta\Sigma - D\right) \ , \\ (\Delta s)_\Lambda &= \frac{1}{3}\left(2\Delta u - \Delta d + 2\Delta s\right)_p \\ &= \frac{1}{3}\left(\Delta\Sigma + 2D\right) \ , \end{aligned} \tag{19}$$

In this case, it is assumed that both the valence and sea quarks are related by $SU(3)$ flavor symmetry. As an example of this procedure, we present in Table IV the results for the spin content of the Λ hyperon in the Ellis-Jaffe-Sehgal analysis (EJS) and another one based on the DIS results for the proton (DIS). In the former, it is found that the up and down quarks are negatively polarized and that the total contribution from the quarks and antiquarks to the Λ spin is reduced to $\Delta\Sigma = 0.586$ [52]. An analysis of the experimental DIS data for the proton [40, 41] in combination with Eq. (19) shows that the strange quarks (and antiquarks) carry about 65 % of the Λ spin, while the up and down quarks (and antiquarks) account for a negative polarization of –32 %. The negative polarization of the up and down quarks is confirmed by different theoretical studies, such as the chiral quark-soliton model [53], lattice QCD [54] and QCD sum rules [55]. It has been pointed out, that $SU(3)$ symmetry breaking effects in hyperon β decays may reduce the negative polarization [53, 56].

Another assumption about the sea sometimes used in the literature is that the sea polarization is the same for all octet baryons, whereas the valence quarks are related by $SU(3)$ symmetry [52, 61]. However, experimental information on the violation of the Gottfried sum rule [62] and the suppression of the polarized strange quark momentum contribution with respect to that of the nonstrange quarks [63], shows that the sea quark distributions depend on the valence quark content in a nontrivial manner.

In the unquenched quark model there is no need to make additional assumptions about the nature of the sea. The valence quarks are related by $SU(3)$ flavor symmetry, but the flavor symmetry is broken by the the sea quarks (see Eq. (1)). Therefore, the $SU(3)$ flavor symmetry relations in Eq. (19) do not hold in the unquenched calculations. Table IV shows that, just as for EJS and DIS, the unquenched quark model gives rise to a negatively polarized sea of up and down quarks, but its results are a lot closer to the CQM values than those of EJS and DIS. The present analysis of the spin content of the proton and the Λ hyperon shows in an explicit way the importance of $SU(3)$ breaking effects.

VI. SUMMARY AND CONCLUSIONS

There is ample experimental evidence for the importance of sea quarks in the structure of hadrons. In this paper, we discussed an unquenched quark model for baryons which incorporates the effects of quark-antiquark pairs. The quark loops are taken into account via a $^3P_0$ pair creation model. The ensuing unquenched quark model is valid for any baryon (or baryon resonance), includes all light flavors of the pairs ($u\bar{u}$, $d\bar{d}$ and $s\bar{s}$), and can be used for any CQM, as long as its wave functions are expressed in a harmonic oscillator basis.

Obviously, the unquenching of the quark model has to be done in such a way that it preserves the phenomenological successes of the CQM. As an example, we showed that, after renormalization of the quark magnetic moments, the inclusion of quark-antiquark pairs does not change the good CQM results for the magnetic moments of the octet baryons. In a similar way, one has studied the effects of hadron loops on the OZI hierarchy [25], self-energies [64, 65] and hybrid mixing [66].

In an application of the unquenched quark model to the spin of the proton and the Λ hyperon, it was found that the inclusion of $q\bar{q}$ pairs leads to a relatively large contribution of orbital angular momentum to the spin of the proton (~ 32 %) and a somewhat smaller amount for Λ (~ 15 %). The difference between these numbers is an indication for the breaking of $SU(3)$ flavor symmetry in the unquenched quark model. The valence quarks are related by flavor symmetry, but the contribution of the sea quarks is determined by the $^3P_0$ coupling between the valence quarks and the higher Fock states without any additional assumption. The contribution of strange quarks to the proton spin is found to be very small, in agreement with results in the cloudy bag model and the NJL model. The relative contribution of up and down quarks $\Delta u/\Delta d$ is reduced from -4 in the CQM to -2.6. For the Λ hyperon we found a small contribution of a negatively polarized sea of up and down quarks, in qualitative agreement with other studies. The spin content of Λ is dominated by the strange quark spins. The results of the unquenched quark model for the spin content of Λ are much closer to the CQM values than that of the proton. In order to be able to make a more detailed comparison with experimental data, one has to include the effects of relativity and evolve the scale dependent quantities to the experimental scale. The present results represent a first step. Relativistic calculations are underway in front form and point form dynamics [67].

The sum over intermediate baryon-meson states is carried out explicitly and includes all possible intermediate states: singlet, octet and decuplet baryons and pseudoscalar and vector mesons as well as their orbital ex-

citations up to any oscillator shell. The convergence of the sum depends on the quantity one is interested in. For the orbital angular momentum, the convergence is very rapid, since the sum is dominated by the contribution of the pions. On the other hand, for the quark spins the sum over intermediate states is dominated by the contribution of the vector mesons and many oscillator shells have to be included before reaching convergence.

The main idea of this paper was to present an unquenched quark model in which the effects of quark-antiquark pairs are introduced explicitly, and which offers the possibility to study the importance of $q\bar{q}$ pairs in hadrons in a systematic and unified way. To the best of our knowledge, these are the first explicit calculations of the sea contributions in the quark model. The present results for the magnetic moments and the spin content of octet baryons in combination with preliminary results for the flavor asymmetry of the nucleon [29] are very promising and encouraging. We believe that the inclusion of the effects of quark-antiquark pairs in a general and consistent way, as suggested here, may provide a major improvement to the constituent quark model which increases considerably its range of applicability.

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